Condorcet/Schulze Voting in Single-Winner Districts

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Summary. I recommend that the House of Commons of Canada should be elected in single-winner districts exclusively. I recommend that the House should be elected by Condorcet/Schulze voting. Condorcet/Schulze voting is a ranked ballot method. Therefore, Condorcet/Schulze voting is within the scope of the campaign promise of the Liberal Party to study and consider ranked ballots and proportional representation.

1. Condorcet Voting

When Condorcet voting is being used, then each voter gets a complete list of all candidates and ranks these candidates in order of preference. The individual voter may give the same preference to more than one candidate and he may keep candidates unranked. When a voter does not rank all candidates, then this means (1) that this voter prefers all ranked candidates to all unranked candidates and (2) that this voter is indifferent between all unranked candidates.

Suppose $N[a,b]$ is the number of voters who prefer candidate a to candidate b. When $N[a,b] > N[b,a]$, then we say that candidate a beats candidate b *directly*.

A *Condorcet winner* is a candidate a who beats every other candidate b *directly*. It can happen that there is no Condorcet winner. *Condorcet voting* means that, when there is a Condorcet winner, then this candidate must be the unique winner.

2. Schulze Voting

Schulze voting comes into play when there happens to be no Condorcet winner. Schulze voting then takes *indirect* defeats into account. We say that candidate a beats candidate b *indirectly*, when candidate a beats candidate b *directly* or when candidate a beats someone who beats candidate b *directly* or *indirectly*.

To use a more formal definition: Suppose $c(1),...,c(n)$ is a path from candidate $a \equiv c(1)$ to candidate $b \equiv c(n)$. The *strength* of this path is the smallest of the margins of victory of the $n-1$ pairwise elections in this path. So the strength of $c(1),...,c(n)$ is the minimum of $N[c(i),c(i+1)] - N[c(i+1),c(i)]$ with $i = 1,...,(n-1)$. We say that candidate a beats candidate b *indirectly* when the strongest path from candidate a to candidate b is stronger than the strongest path from candidate b to candidate a.

A *Schulze winner* is a candidate a who beats every other candidate b *indirectly*. It can be proven that there is always a Schulze winner. *Schulze voting* means that the winner must be a Schulze winner.

Determining the Schulze winner is a bit complicated. However, Schulze voting is not justified by its algorithm to calculate the winner. Rather, it is justified by the large number of beneficial properties it has. It can be proven that Schulze voting satisfies all important criteria that are compatible with the Condorcet criterion (e.g. anonymity, neutrality, homogeneity, transitivity, resolvability, Pareto, reversal symmetry, monotonicity, independence of clones, majority criterion, Smith criterion, Schwartz criterion, prudence).

Condorcet/Schulze voting is currently the most wide-spread Condorcet voting method. It is used by many software projects (e.g. Debian, Ubuntu, Gentoo, OpenStack, Software in the Public Interest), by the Pirate Party in over a dozen countries, by dozens of other organizations (e.g. Five Star Movement of Italy, German Association of Pediatricians, Albert Ludwig University of Freiburg), and by the city of Silla in Spain.

3. Example

There are 4 candidates and 21 voters.

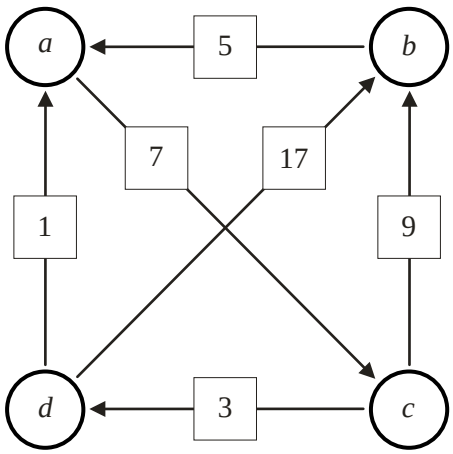
8 voters prefer *a* to *c* to *d* to *b*.
2 voters prefer *b* to *a* to *d* to *c*.
4 voters prefer *c* to *d* to *b* to *a*.
4 voters prefer *d* to *b* to *a* to *c*.
3 voters prefer *d* to *c* to *b* to *a*.

The pairwise matrix *N* looks as follows:

	N[*,*a*]	*N*[*,*b*]	*N*[*,*c*]	*N*[*,*d*]
N[*a*,*]	---	8	14	10
N[*b*,*]	13	---	6	2
N[*c*,*]	7	15	---	12
N[*d*,*]	11	19	9	---

The pairwise matrix can also be written as a graph. When $N[i,j] > N[j,i]$, then there is a link from candidate *i* to candidate *j* of strength $N[i,j] - N[j,i]$.

The corresponding graph looks as follows:



There is no Condorcet winner. Therefore, Schulze voting comes into play.

The strongest path ...
... from candidate *a* to candidate *b* is $a \xrightarrow{7} c \xrightarrow{9} b$ with a strength of 7.
... from candidate *a* to candidate *c* is $a \xrightarrow{7} c$ with a strength of 7.
... from candidate *a* to candidate *d* is $a \xrightarrow{7} c \xrightarrow{3} d$ with a strength of 3.
... from candidate *b* to candidate *a* is $b \xrightarrow{5} a$ with a strength of 5.
... from candidate *b* to candidate *c* is $b \xrightarrow{5} a \xrightarrow{7} c$ with a strength of 5.
... from candidate *b* to candidate *d* is $b \xrightarrow{5} a \xrightarrow{7} c \xrightarrow{3} d$ with a strength of 3.
... from candidate *c* to candidate *a* is $c \xrightarrow{9} b \xrightarrow{5} a$ with a strength of 5.
... from candidate *c* to candidate *b* is $c \xrightarrow{9} b$ with a strength of 9.
... from candidate *c* to candidate *d* is $c \xrightarrow{3} d$ with a strength of 3.
... from candidate *d* to candidate *a* is $d \xrightarrow{17} b \xrightarrow{5} a$ with a strength of 5.
... from candidate *d* to candidate *b* is $d \xrightarrow{17} b$ with a strength of 17.
... from candidate *d* to candidate *c* is $d \xrightarrow{17} b \xrightarrow{5} a \xrightarrow{7} c$ with a strength of 5.

The unique Schulze winner is candidate *d* because, for every other candidate *x*, the strongest path from candidate *d* to candidate *x* is stronger than the strongest path from candidate *x* to candidate *d*.

References

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